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Scaling Laws of Paleoindian Projectile Point Design

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Abstract
Across late Pleistocene North America, Paleoindians designed a variety of projectile point styles that vary in form over time and space, but also share similar performance requirements and a common technological history. In order for projectile points to be effective components of hunting weaponry, at a minimum they must be engineered to be both effective at penetrating prey and robust to failure. These universal performance criteria suggest the design space underlying the diversity of projectile point styles we observe in the archeological record must be highly constrained by engineering principles. Here, using a large sample of complete points ($n = 2360$) from 16 Paleoindian projectile point types we examine the diversity of Paleoindian projectile point designs using a combination of engineering principles, allometric scaling theory, and statistical models. First, we define the design space of Paleoindian projectile points by examining the dimensions of length, width, and thickness that define the basic three-dimensional volumes of the points and their size allometries. Second, we derive three hypotheses for the optimal allometric design of points from first principles of engineering that quantify design trade-offs between robustness and penetration capability. We then test the predictions of the hypotheses with data and show that the empirical observations match the theoretical predictions in almost every case. Our analysis shows that Paleoindian projectile points are engineered to resist breaking while maximizing penetrating capabilities, and we show the engineering principles behind this optimization. We also show that within these design criteria, some types are designed to emphasize penetration, whereas others emphasize robustness. These results demonstrate that once bifacial point traditions emerge in Paleoindian North America there is a high retention of design, engineering, and performance criteria throughout the Paleoindian record, and probably beyond.

Keywords Engineering · North America · Allometry · Optimization · Types · Raw materials

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Introduction

Paleoindian projectile points are stone bifaces inferred to have been hafted to the end of spears or darts to be used primarily as projectile weapons (see Hutchings 2015). Paleoindian projectile points in North America were used from at least ~13,500 calendar years before present (hereafter calBP) until ~10,000 calBP, although this range varies by region (Anderson et al. 2015; Holliday 2000; Justice 1987, 2002a, 2002b; Lothrop et al. 2016). Projectile points were manufactured from a variety of fine-grained stone raw materials, and most were capable of being reused multiple times. Based on ethnographic analogy (*e.g.*, Ellis 1997) and archeological context (*e.g.*, Buchanan et al. 2011) researchers infer that projectile points were used to hunt game animals, though use-wear suggests were also used as cutting tools, and undoubtedly served a variety of other purposes over their use-lives (Kay 1996; Smallwood 2015). As such, the design and engineering of projectile points balanced manufacturing costs with performance criteria, including portability, aerodynamics, robustness, and the capability to penetrate the bodies of prey animals causing either death or sufficient damage to allow the injured animal to be tracked and killed (Bleed 1986; Christenson 1986; Hughes 1998; Loendorf et al. 2018; Pargeter 2007; Salem and Churchill 2016; Smith 2015). Much experimental work with stone point replicas has explored the role of point shape (Cheshier and Kelly 2006; Odell and Cowan 1986), size (Maguire et al. 2020), raw material (Loendorf et al. 2018), flaking (Lowe et al. 2019), and fluting (Story et al. 2019; Thomas et al. 2017) in the longevity and durability of a point.

Despite these universal performance criteria Paleoindian projectile points vary in form over time and space, and this variation forms the basis of Paleoindian projectile point typologies in the archeological record. Although Paleoindian projectile points are often described as "lanceolate" (*i.e.*, generally long, broad, and tapering toward the tip) this term belies a wide diversity of styles and manufacturing techniques where individual types may be gracile or robust, long or short, with concave or convex edges, fluted or unfluted, manufactured from bifaces or flakes, and so on. Often there is considerable diversity within a particular type leading to much debate over systematics. For example, it is commonly debated whether Midland points are simply unfluted Folsom points (Amick 1995; Jennings 2016) and Goshen points have recently been argued to be Plainview points (Buchanan et al. 2020). There is also considerable variation generated over the use-life of a point where dimensions necessarily change as points are reused, resharpened, or repaired. Consequently, there are multiple sources of variation in projectile point form at all scales, from design decisions and individual life histories, to adaptive innovations in local environments as well as regional and continental scale phylogenies that constrain the entire evolutionary trajectory of Paleoindian projectile point design (*e.g.*, Buchanan et al. 2012a, b, 2015, 2018; Gingerich et al. 2014; Jennings 2008; Morrow and Morrow 1999; O'Brien et al. 2014, 2016; Shott 2010; Smallwood et al. 2019).

Clearly, Paleoindian projectile points were highly engineered tools manufactured by skilled flintknappers who understood that in order to achieve high levels of performance, specific design decisions were required. As such, there is very little that is random about the design of a projectile point. Within particular traditions in time and space, projectile points conformed to a set of design criteria, the effect of which was to produce spatial and temporal patterning in the archeological record and the variation between different types reflects the different design choices made by groups of flintknappers belonging to different cultural traditions in time and space. The question of interest in this paper is whether variations within and among projectile point types result from locally independent design decisions, or whether individual projectile point types are local variants of global design criteria. If so, how closely do local designs conform to global designs and what were those design principles?

One particularly productive approach to understanding the design, engineering, and performance of stone projectiles is through carefully controlled and standardized experiments in the laboratory, where the physics of the projectiles and the forces acting on them can be manipulated in highly controlled settings, and responses can be measured and repeated (Loendorf et al. 2018; Lowe et al. 2019; Maguire et al. 2020; Story et al. 2019; Thomas et al. 2017). Another productive approach is the theory-driven analysis of data. In this paper, we examine the statistics of a large sample of complete projectile points from the archeological record given their volumetric allometries and the physics of projectile point performance. This sample provides insight into the range of projectiles actually in use at any one time during the Paleoindian period, as each individual point entered the archeological record while still complete though at varying stages of their use-life. Therefore, all the points in this sample were first designed in a certain time and place to meet a set of functional and aesthetic requirements, manufactured to meet those design criteria, and then used over varying lengths of time before being discarded or lost. Importantly, the sequence of use, repair, and reuse reflects the selective filtering of performance criteria on projectile point form, and the samples we examine are those that retained viability by remaining complete.

Understanding the design principles of projectile points is ideally suited to allometric scaling analysis, a statistical method that specifically asks how the dimensions of an object vary with size (Bonner 2011; Calder 1984; McMahon and Bonner 1983; Schmidt-Nielsen 1984; West 2017). Moreover, allometric scaling is directly related to principles of Euclidean geometry, which are well-understood and axiomatic. If the dimensions of an object remain invariant under size transformation, then this scaling symmetry is said to be geometrically similar, or isometric (Fig. 1A). If the dimensions change under size transformation, the scaling is allometric (Fig. 1B). Therefore, we are interested in how the linear dimensions (length, width, and thickness) of a projectile point change with size (mass or volume). In Euclidean geometry a 2-dimensional object, such as an area, a, consists of two linear dimensions, $a \propto l^2$, and a 3-dimensional object, such as a mass, m or volume, v consists of three linear dimensions, $m = v \propto l^3$. These Euclidean constraints mean that under invariant size transformation (*i.e.*, isometry) a 3-dimensional object and a linear dimension are related as $l \propto m^{\beta_l}$, where $\beta_l = 1/3$, an area as $a \propto m^{\beta_a}$ where $\beta_a = 2/3$, and a volume as $v \propto m^{\beta_v}$ where $\beta_v = 1$. The scaling exponent $\beta_x = d \ln x / d \ln v$ is an elasticity that captures the relative change in dimension to a relative change in volume.

Projectile points are 3-dimensional objects whose three linear dimensions are described by a length, l; a width, w; and a thickness, t, where by definition $l > w > t$. Further, by the definition above, these three linear dimensions completely define the 3-dimensional volume, v, of a point, no matter the shape. If dimensions are isometric to volume (*i.e.*, their dimensions are invariant to size transformation) then:

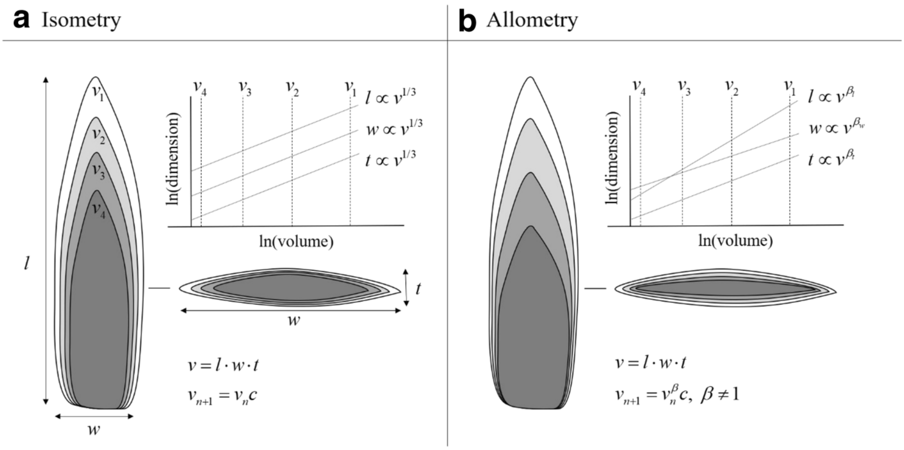

Fig. 1 Isometric *vs* allometric scaling in projectile points. If projectile point volumetric shape scales isometrically with size then points of different sizes are linearly rescaled versions of each other, and so dimensions are invariant to size transformation (**A**). If projectile point volumetric shape is allometric on the other hand, then points of different sizes are nonlinearly rescaled versions of each other, and so dimensions vary with size transformation (**B**)

$$l = c_l v^{1/3} \tag{1}$$

$$w = c_w v^{1/3} \tag{2}$$

$$t = c_t v^{1/3} \tag{3}$$

where c_x are dimensional constants. Because these three linear dimensions constitute a volume the sum of the three exponents must equal unity, which places an important set of constraints on projectile point allometry; that is $\beta_l + \beta_w + \beta_t = 1$. As such, a non-Euclidean scaling (*i.e.*, $\beta_x \neq 1/3$) in any of the linear dimensions must be compensated for in the scaling of at least one of the other dimensions. We will show later how this leads to a constrained optimization of allometric exponents. These Euclidean principles then provide a robust theoretical framework to examine allometries, and deviations from isometry. The statistical question at hand is how does the scaling behavior of projectile point dimensions vary with volume compared with the Euclidean null expectations and what does this imply about their design?

Materials and Methods

The Data The sample we use in our analysis consists of 2360 complete points from 16 Paleoindian projectile point types from the lower 48 states of the USA and Chihuahua, Mexico (these data can be found in the Supplementary Data). We focus on projectile point dimensions including length, width, thickness, and volume, and additional design features including cross-sectional area and sectional density, as well as various measures of robustness all of which are derived from the three linear dimensions. The purpose of our study is to explore the allometric scaling of these dimensions in relation to mass (*i.e.*, volume or weight). We then compare scaling relationships within our sample controlling for type and raw material using fixed-effects models.

Selection Criteria Our dataset is derived primarily from the Paleoindian Database of the Americas (PIDBA; Anderson et al. 2010) and publications that include measurements of Paleoindian points. The Paleoindian points included in our study are all complete enough to have reliable measures of length, width, and thickness. We recorded point type if a classification was reported in PIDBA or in the literature. The PIDBA sample includes 24 point types. Eight of these types have samples of 100 or more points and represent about 72% of the overall sample and include Clovis, Eastern Clovis, Dalton, Folsom, Greenbrier, Cumberland, Beaver Lake, and Harpeth River. The PIDBA dataset is comprised of contributed data, and projectile point types are not reassigned or evaluated by experts. This means that many of the less well-represented types in particular are not well defined and are sometimes "collector" types. We excluded the eight types from the database that have a sample size of less than 20 specimens. We also excluded all specimens not assigned to a type. For clarity, replication, and consistency, we followed the type designations used by PIDBA. It should be noted that some of the type designations in this database undoubtedly would be reclassified, and perhaps combined or split if a comprehensive re-analysis of typology was undertaken by experts. Other types, such as Northumberland, are collector categories, and so are not well defined or recognized in the literature. However, we include all types that meet our minimum sample size criteria as the results of our analyses are directly relevant to considering how types such as these compare to all others across various dimensions. Moreover, we show here that projectile point type designations are of only minor relevance in the general patterns of allometric scaling in Paleoindian projectile point design.

We also recorded the type of stone raw material if reported. As with the type designations, the raw material categories are based on identifications made in the PIDBA database and the published literature. We assume that raw material identifications were based primarily on visual inspection and so have different levels of error; *i.e.*, some categories are easily identified (*e.g.*, obsidian), and others are less so (*e.g.*, distinguishing chert from chalcedony and jasper). Ten different types of stone were used to make the points in our sample (Table 1). Overall, the most common raw material was chert (79%), whereas the other identified raw materials comprise less than 4% each. For about 5% of the sample the raw material could not be determined or was not identified, and so labeled as "indeterminate."

Statistical Analyses

All continuous data were right-skewed and normalized through log-transformation (see below). In addition, the theory of allometry is based on the mathematics of power functions (Eqs. 1, 2, and 3), which are linearized by log-transforms of both size (Eq. 4),

Table 1 Raw material type identifications for the points in the sample as reported in PIDBA and the published literature

Raw material type	Number in sample
Agate	19
Chalcedony	65
Chert	2103
Jasper	87
Metavolcanic	96
Obsidian	7
Quartz	64
Quartzite	39
Rhyolite	19
Sandstone	28
Silicified wood	11
Indeterminate	130
Total	2360

$$\ln l_x = \ln c_x + \beta_x \ln v \tag{4}$$

where β_x is now the slope of a straight line, which can be modeled using regression or linear models (both of which we use here). The appropriate scale of any allometric analysis is thus logarithmic rather than linear, because the mechanisms that generate variation in allometry are multiplicative, not arithmetic. We estimate scaling parameters (and their statistics) across the entire data set using ordinary least squares (OLS) regression analyses on the logarithmically transformed variables. To examine the effects of typology and to control for variation in raw materials we then use general linear models with random intercepts and random slopes, where projectile point type is a fixed factor. All analyses are conducted in R version 3.5.1 (R Development Core Team 2018) and RStudio version 1.1.456 (RStudio 2018). All data, results, and code are provided in the Supplementary Material, and so all aspects of our study can be easily replicated.

Results

Figure 2 shows that all three linear dimensions of log-transformed data are well-fit by normal distributions, and so the data are approximately log-normal on an arithmetic scale. Note that because of the large sample size, the central limit theorem states that a random sample from any of these distributions will be normally distributed, even in the less-well-behaved Fig. 2C. As such, we proceed by using parametric statistics and linear models on the log-transformed data.

The violin plots of these dimensions by projectile point type in Fig. 3 show that while there is clear variability among types, there is also remarkable consistency across

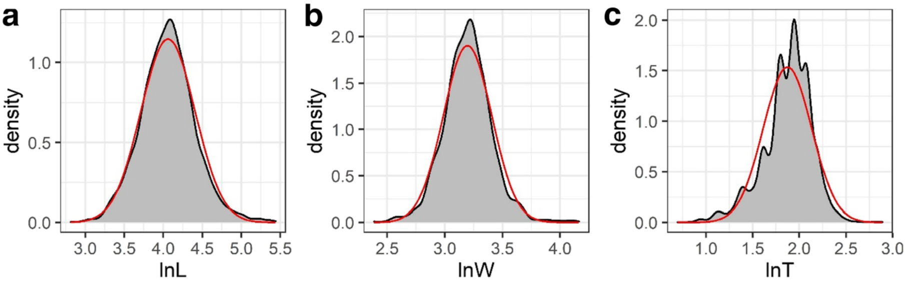

Fig. 2 Density distributions of point width, length, thickness, and volume. All distributions are approximately normally distributed on the log scale (red curves)

the sample, suggesting that point dimensions are constrained and central tendencies are well defined (Fig. 3). We control for this variation below using general linear models (GLMs) where point type is a fixed effect.

Defining the Paleoindian Projectile Point Design Space

First, we consider the design space of Paleoindian projectile point types. Figure 4 is a ternary plot of the theoretical 3-dimensional morphospace that defines all possible

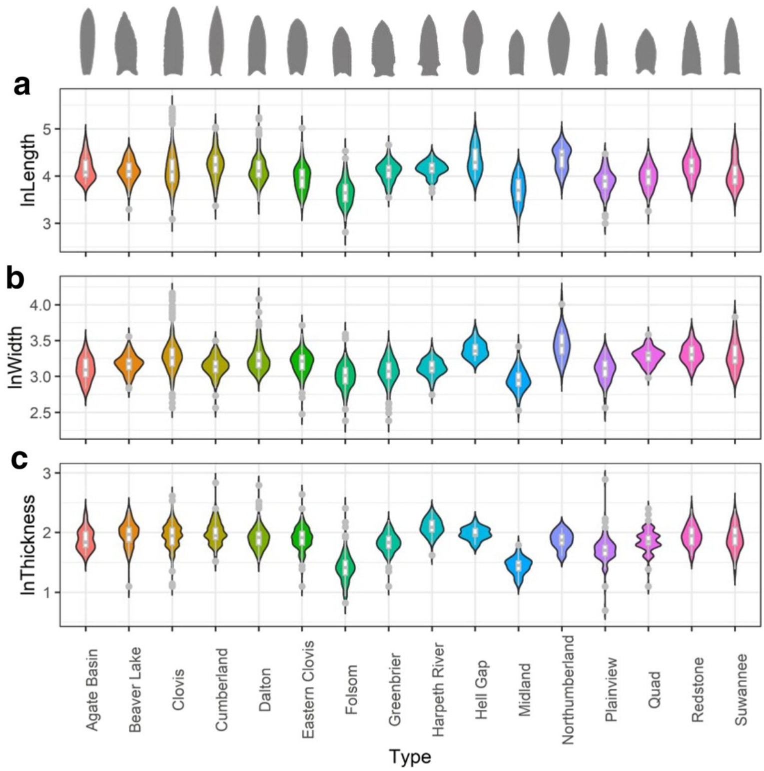

Fig. 3 Violin plots of **A** width, **B** length, and **C** thickness by projectile point type on a logarithmic scale with the typical shapes of each type along the upper x-axis (not to scale)

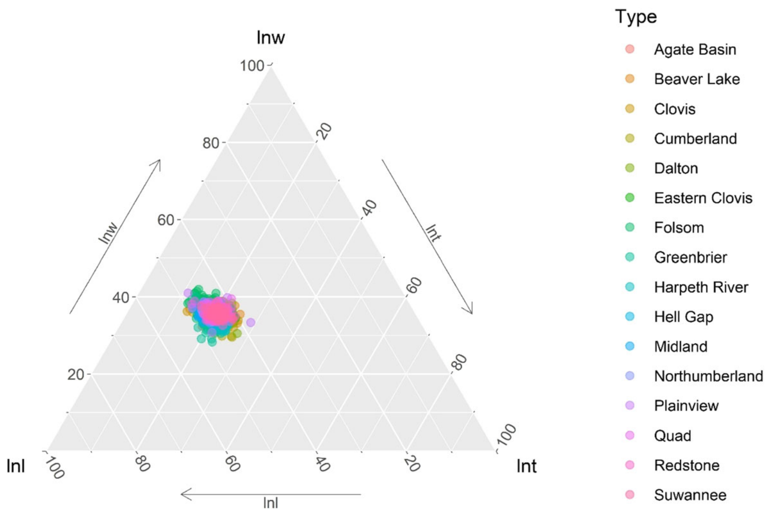

Fig. 4 Ternary plot of the linear dimensions showing tight clustering of the 2360 projectile points over the 16 types in design space

combinations of the three linear dimensions of projectile points. There are three main observations to be drawn from Fig. 4: First, the dimensions of all 2360 points across the 16 types collapse onto a well-defined cluster. Second, the cluster onto which they collapse is confined to a very restricted region of the total available design space, and third, the clustering is not centered but displaced along the length and thickness axes. This tight clustering shows no dispersion along any of the three axes indicating strong selective gradients on all three dimensions. These selective gradients are combination of physical engineering constraints, design principles, performance criteria, and a shared evolutionary history. The displacement along the length and thickness axes suggests that these two dimensions are correlated.

Paleoindian Point Mass and Volume Equivalence

Length, width, and thickness are the three linear dimensions of a projectile point's volume, which we define as $v = w \times l \times t$. Note that these dimensions could be used equivalently to define the volume of any 3-dimensional shape of a projectile point (*i.e.*, ellipsoid, oval, rhomboid, and octahedron) and so this specific definition of volume leads to no loss of generality. In our dataset, only 22% of projectile points have mass measurements (519 of 2360) and these are not a random sample as they come from only six types (Clovis, Dalton, Eastern Clovis, Folsom, Midland, and Plainview). Figure 5 shows that volume is a linear function of mass, where $m = m_0 v^{1.01}$ (OLS: $F_{517} = 14{,}355$, $R^2 = 0.97$, $p < 0.0001$), and so the mass of any point is simply the total volume, v, rescaled by the specific volume, m/v. Further we show in Table 2 that neither this constant nor the exponent is affected by raw material type. In other words, for a given volume, the mass of a projectile point does not vary by raw material type. This mass–volume equivalence allows us to substitute mass by volume and so examine the allometry across the entire dataset of 2360 points and 16 types.

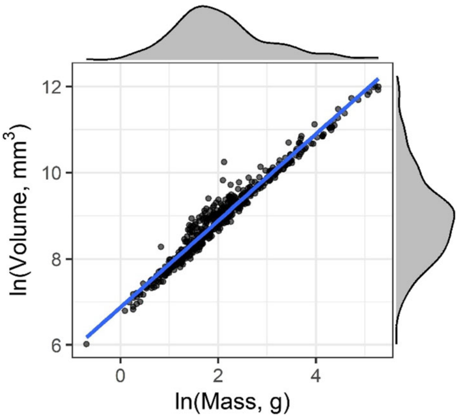

Fig. 5 Paleoindian point volume as a function of weight. Volume ($v = w \times t \times l$) increases linearly with mass

Table 2 Results of a GLM of projectile point volume by mass and raw material showing that while volume is a linear function of mass, raw material type has no effect on the mass-volume relationship *

Predictors	Estimates	CI	*p*
Intercept	2.44	2.25–2.63	< 0.001
ln*M*	1.01	0.91–1.12	<0.001
RM (Chalcedony)	− 0.00	− 0.20 – 0.20	0.999
RM (Chert)	0.06	− 0.13 – 0.25	0.523
RM (Indeterminate)	2.24	− 4.63 – 9.11	0.522
RM (Obsidian)	− 0.02	− 0.43 – 0.40	0.943
RM (Quartz)	0.08	− 0.14 – 0.31	0.467
RM (Quartzite)	0.10	− 0.14 – 0.33	0.420
RM (Silicified Wood)	0.01	− 0.25 – 0.27	0.939
ln*M* * RM (Chalced.)	− 0.01	− 0.12 – 0.11	0.912
ln*M* * RM (Chert)	0.00	− 0.10 – 0.11	0.943
ln*M* * RM (Indetermin.)	− 1.64	− 6.75 – 3.47	0.529
ln*M* * RM (Obsidian)	0.06	− 0.27 – 0.38	0.726
ln*M* * RM (Quartz)	− 0.05	− 0.21 – 0.10	0.498
ln*M* * RM (Quartzite)	− 0.08	− 0.27 – 0.12	0.434
ln*M* * RM (Sil. Wood)	− 0.03	− 0.33 – 0.26	0.824
N	519		
R^2/R^2 adjusted	0.968/0.967		

*Analysis of variance table: response: lnV

	Df	Sum Sq	Mean Sq	*F* value	Pr(> *F*)
lnM	1	495.98	495.98	15185.6323	< 0.0001
factor(RM)	7	1.35	0.19	5.9170	< 0.0001
lnM:factor(RM)	7	0.08	0.01	0.3565	0.9269
Residuals	503	16.43	0.03		

Allometric Scaling of Paleoindian Projectile Point Performance and Robustness

A central component of the performance of a projectile is mass, as mass is a central component of the generation of force, F. In Newton's second law, $F = ma$, where m is mass and a is acceleration (*i.e.*, the rate of change in velocity, g). In the technologies of traditional hunter–gatherer societies, the acceleration of a projectile is a function either of the biomechanics, strength, and skill of the hunter and/or the method of propulsion employed (*i.e.*, spear, atlatl, or bow) (Whittaker et al. 2017). The majority of the total mass of a projectile is provided by the shaft and so the point's primary role is to penetrate the body cavity of a prey animal, and so the shape of the point is particularly important.

The physics of penetration starts with momentum, $p = mg$, where m is mass and g is velocity. As a projectile strikes a prey animal 100% of momentum is transferred from the projectile to the prey, and the physics of this impact is such that heavier projectiles will penetrate more deeply than lighter projectiles. This is because although heavier objects travel at slower velocities than lighter objects, the heavier projectile will decelerate at a slower rate while passing through tissues because tissue resistance increases as the square of velocity (Ashby 2018). In addition to momentum, penetration depth is a function of projectile point shape. Smooth-edged, broad-bladed projectiles with low profiles maximize available momentum and therefore maximize penetration by minimizing tissue resistance. Penetration depth is thus also a function of the cross-sectional area of a projectile (Fig. 6A),

$$a = \frac{wt}{2} \tag{5}$$

and the sectional density, which is the mass divided by the cross-sectional area, $s = m/c \approx v/a$. The greater the sectional density, the higher the ratio of mass to cross-sectional area, which minimizes tissue resistance at the tip of the projectile and so maximizes penetration (Ashby 2018; also see Hughes 1998; Sisk and Shea 2009). Further, smooth broad blades increase the mechanical advantage of a projectile by minimizing the profile and resistance while maximizing the transmission of force (Ashby 2018). In addition, the longer and straighter the slope of the blade, the greater the mechanical advantage. Therefore, to maximize penetration depth Paleoindian flintknappers should produce projectile points that are long, broad, and flat, and indeed, most Paleoindian projectile points follow this general model. Cross-sectional areas are minimized by reducing the thickness and thus minimizing the ratio of thickness to width to create a flat and broad point, thus offering the least tissue resistance. Minimizing relative thickness necessarily decreases the sectional density of the point as it reduces both volume and cross-sectional area; however, this reduction will likely have negligible effects on penetration depth as the majority of the total mass of a projectile comes from the shaft, not the point.

However, minimizing the thickness of a point necessarily reduces robustness. In engineering, the inability of an object (such as a beam, or a projectile point) to resist a critical load results in buckling, or breakage (Fig. 6B). The resistance to buckling is determined by the slenderness ratio,

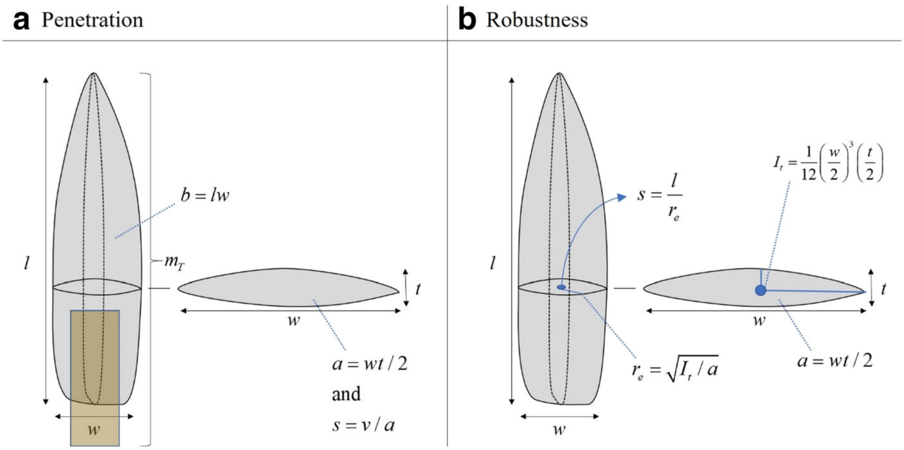

Fig. 6 The calculations involved with penetration and robustness in projectile points from the three linear dimensions of width, thickness, and length

$$s = \frac{l}{r_e} \tag{6}$$

which is the ratio of point length to its effective radius (or the radius of gyration), which is

$$r_e = \sqrt{\frac{I_t}{a}} \tag{7}$$

Here, a is the cross-sectional area, as defined above, and I_t is the second moment of inertia, which is effectively the ability of a point to bend through the thickness plane before breaking. In an asymmetrical cross-sectional area, the second moment of inertia along the thickness plane is $I_t = wt^3/12$ (see Fig. 6B). It then follows that in order to resist buckling the two components of the slenderness ratio (Eq. 6) scale as

$$r_e \propto l^{2/3} \tag{8}$$

Therefore, to maximize the resistance to breaking, projectile points must maintain a constant allometry between thickness and length across the range of volume;

$$t \propto l^{2/3} \tag{9}$$

The importance of thickness relative to length was highlighted empirically in experiments by Cheshier and Kelly (2006), where they examined projectile point durability, which they defined as the number of times an experimental point could be fired at a target before failure.

From this understanding of the physics of penetration and robustness we can then derive several predictions that can be tested with our data.

Optimization of Projectile Point Allometries

Following from above, let us assume a Paleoindian flintknapper will aim to manufacture a point of certain dimensions using locally specific design criteria that meet the engineering constraints of performance and robustness. Over time, as a point is used and/or damaged, but remains intact, the point will require repair or resharpening, which in a reductive technology, necessarily requires the loss of mass and therefore volume. Resharpening is not random with respect to dimensions and will usually entail the reduction of maximum length first, thickness second, and width, third. However, as the projectile point loses mass a flintknapper will attempt to reshape the point in such a way as to maintain performance and robustness by preserving the original dimensional allometries as much as possible.

From this simple model we develop an optimization to generate expectations of projectile point allometry given straightforward performance and robustness requirements. We first observe that the allometric scalings of the three linear dimensions must sum to unity: $\sum\beta = \beta_l + \beta_w + \beta_t = 1$; therefore, dimensional allometries must trade-off with each other (Eqs. 1, 2, and 3). Further, in order to maintain robustness, the ratio of two of the allometries, thickness and length, must remain constant at $\beta_t/\beta_l = 2/3$ (Eqs. 8 and 9). Within the robustness constraint, thickness will be minimized relative to width in order to create a flat, broad blade that will maximize the available mechanical advantage. As the point is resharpened this mechanical advantage will be maintained as mass is lost by preserving the ratio, $\beta_w/\beta_t = 1$, which implies $\beta_w = \beta_t$. We can then solve numerically for the optimal allometries of projectile point dimensions with respect to volume that simultaneously maximize penetration depth and robustness (Fig. 7). In specific, we ask what is the optimal length-volume allometry, β_l, that meets the above conditions of robustness $\beta_t/\beta_l = 2/3$ and the preservation of the cross-sectional ratio $\beta_t/\beta_w = 1$. Solving numerically with respect to volume we find $\beta_l^* = 0.43$. Simultaneously, we ask what is the optimal value of the width and thickness allometries that meet these same conditions. Solving numerically with respect to volume we find $\beta_w^* = \beta_t^* = 0.29$. Note that, allowing for rounding error, $\beta_w^* + \beta_t^* + \beta_l^* = 1$.

We then test the predictions of the following three hypotheses using our data set of 2360 points over 16 types to evaluate whether Paleoindian projectile points are optimally designed to maximize penetration depth and robustness:

H_1: The ratio of the allometries of projectile point thickness and length meets the robustness criteria set by the slenderness ratio, in which case the ratio of the allometric exponents of thickness and width with respect to volume is predicted to be $\beta_t/\beta_l = 2/3$.

H_2: The allometries of projectile point width and thickness with respect to volume are optimal given the constraints of robustness, in which case the allometric exponents of width and thickness with respect to volume are predicted to be $\beta_w = \beta_t = \beta_w^* = \beta_t^* = 0.29$.

H_3: The allometry of projectile point length with respect to volume optimizes length conditional on the constraints of the cross section, in which case the scaling exponent is predicted to be $\beta_l = \beta_l^* = 0.43$.

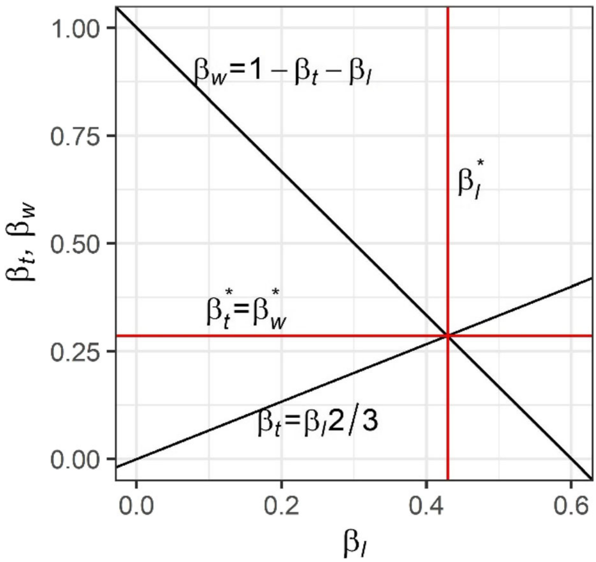

Fig. 7 The optimal trade-offs among the allometric exponents. The rate of change in point length with respect to volume is on the *x*-axis, β_t, and the rates of change in both the width and lengths are along the *y*-axis, β_t and β_w. β_t increases as $\beta_l \times 2/3$ to meet the robustness criteria, and β_w decreases with increasing β_t and β_l because by definition $\beta_l + \beta_w + \beta_t = 1$. Therefore, the point along the *x*-axis that maximizes both β_t and β_w is where these slopes intersect at $\beta_l^* = 0.43$ (the vertical red line), and along the *y*-axis this is $\beta_t^* = \beta_w^* = 0.29$ (the horizontal red line). These then are the predicted allometries for the three linear dimensions of projectile points designed to maximize robustness while also maximizing penetration capability

First, we examine the general allometries of points across the entire data set using OLS regression models. Table 3 provides the parameter estimates for OLS models of the projectile point dimensions and volume and then as a function of mass. Across the entire sample, projectile point length scales with volume as

$$l_{OLS} \propto v^{0.45} \tag{10}$$

Remarkably close to the optimal prediction, projectile point width scales with volume as

$$w_{\mathrm{OLS}} \propto v^{0.25} \tag{11}$$

which is slightly less, but close to the optimal prediction. Projectile point thickness increases with volume as

$$t_{\mathrm{OLS}} \propto v^{0.30} \tag{12}$$

which meets the optimal prediction.

The empirical scaling of robustness from Eqs. 10 and 12 is $\beta_t/\beta_l = 0.30/0.45 = 2/3$, which supports of the predictions of hypothesis 1 and shows that in general, Paleoindian projectile points are engineered to resist breaking. The empirical scaling

Table 3 OLS results of projectile point length, width, and thickness by volume, estimating the scaling coefficients

	ln$L$			ln$W$			ln$T$		
Predictors	Est.	CI	p	Est.	CI	p	Est.	CI	p
Intercept	− 0.04	− 0.11, 003	0.29	0.92	0.86, 0.98	< 0.001	− 0.24	− 0.27, −0.21	< 0.001
B_x	0.45	0.44, 0.46	< 0.001	0.25	0.24, 0.26	< 0.001	0.30	0.29, 0.31	< 0.001
F_{stat}	12,284		< 0.001	5683.5		< 0.001	5014.2		< 0.001
N	2360			2360			2360		
R^2	0.839			0.707			0.680		

of the cross-section from Eqs. 11 and 12 is $\beta_t/\beta_w = 0.30/0.25 = 1.2$, which indicates as projectile points lose volume they have a tendency to decrease faster in thickness than width.

However, the OLS results treat all points as equivalent and do not consider variation in the allometries of projectile points across types. This is important as the particular forms of projectile points vary with type (Fig. 3), which capture discrete variation in design traditions over time and space not accounted for in the simple dimensional allometries. Figures 8 and 9 show the allometric scaling of Paleoindian projectile point width, length, and thickness as a function of volume for each point type using general linear models with random intercepts and slopes. Figure 8 shows that all three dimensions scale tightly with volume: for length (GLM: Adj. $R^2 = 0.87$; $F_{31,2328} = 501.9$; $p < 0.0001$), width (GLM: Adj. $R^2 = 0.80$; $F_{31,2328} = 295.2$; $p < 0.0001$), and thickness (GLM: Adj. $R^2 = 0.76$; $F_{31,2328} = 238.1$; $p < 0.0001$). Detailed statistical results are given in the Supplementary Materials attached to the online version of this paper rather than in the text as the tables are large. Most importantly, Fig. 9 shows that the allometric scaling of all linear dimensions across all types meets the optimal predictions from hypotheses 2 and 3. The only exception is the length allometry of Hell Gap points, which scales significantly steeper with volume than the optimal expectation. However, it should be noted that Hell Gap also has the smallest sample size in the dataset ($n = 24$).

Discussion

Our results show that Paleoindian points were designed both to maximize penetration depth and resistance to breaking. These design principles are evident from the basic allometries of the linear dimensions of volume, all of which are consistent with an optimization model that emerges from the physics of performance criteria. These dimensions define a highly restricted design space within the overall theoretical morphospace of design possibilities (Fig. 4), indicating strong selection on all three dimensions. These selective pressures are design, engineering, and performance criteria, bounded by manufacturing norms and technological traditions inherited across multiple generations of Paleoindian flintknappers. Moreover, all of the 16 types considered here collapse onto the same restricted design space indicating that these

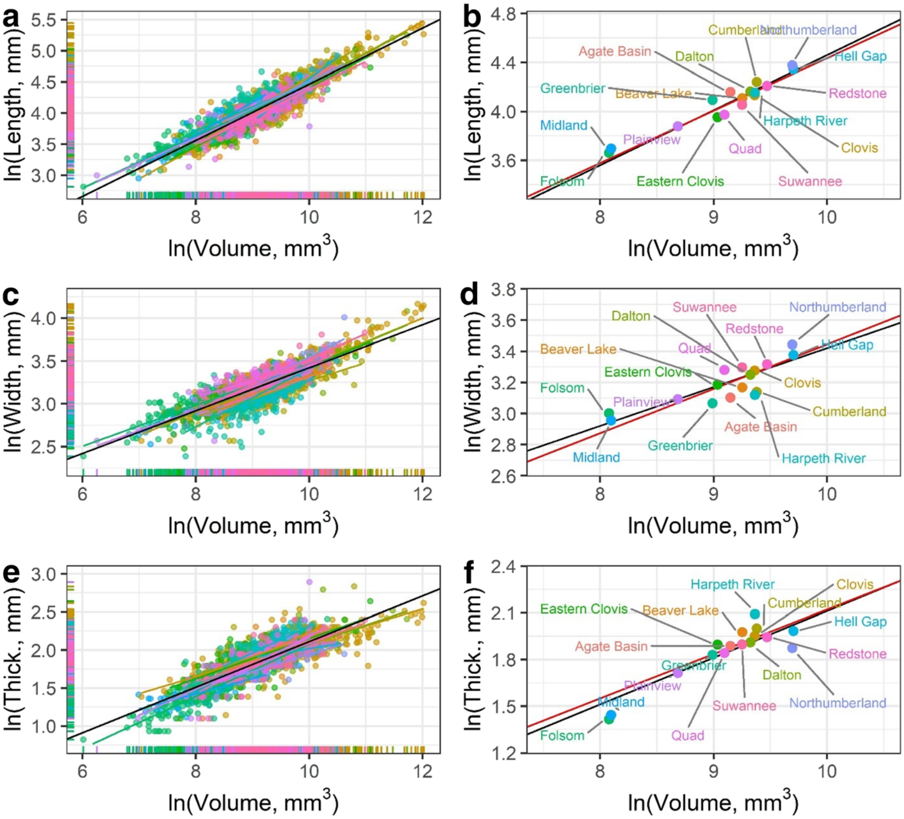

Fig. 8 Allometric scaling of Paleoindian point widths (**A**, **B**), lengths (**C**, **D**), and thicknesses (**E**, **F**) by volume. The plots to the left show the entire data with slopes fitted to each type (see Fig. 8). The plots on the right are the same data but illustrate the relative position of each type in the allometries to the left and show the average dimension *vs.* average volume. The solid black lines are the OLS regression fits across the entire data set from Table 3, and the red lines are the optimal predictions from hypotheses 1–3

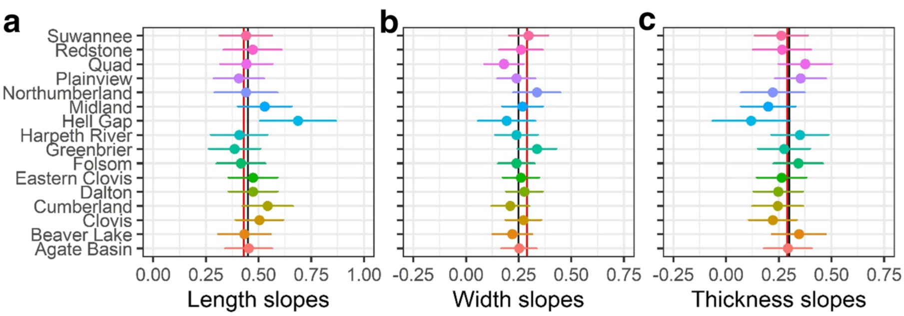

Fig. 9 Results of the fixed effects models for Paleoindian point slopes of length (**A**) width (**B**), and thickness (**C**) by type. The vertical black lines are the slope estimates from the OLS models (Table 3), and the red lines are the optimal predictions from hypotheses 1–3. The horizontal error bars around the individual parameter estimates are the 95% confidence intervals for each type. If the error bars encompass the red lines, then those types are not significantly different from the optimal prediction. In all three plots, the only type significantly different from the optimal predictions in any of the dimensions is the length allometries of Hell Gap points (**A**). We provide pairwise marginal interaction statistics that compare all the slopes in three plots in the Supplementary Material

design principles were common to all 16 Paleoindian traditions. If these 16 traditions are a random sample of all Paleoindian projectile point types, this result suggests that design principles are universal. While the form, shape, and manufacturing techniques employed in the production of projectile points varied over time and space to the extent that Paleoindian archeologists (and presumably Paleoindian flintknappers) recognize the spatiotemporal patterning of types, the selective gradients on performance were so strong that the underlying design criteria were invariant across the continent over thousands of years.

The physics of projectile point penetration shows that long, broad, flat-faced blades maximize the transfer of momentum from projectile to prey by maximizing mechanical advantage and thus penetration depth. Broad flat-faces are well-recognized characteristics of many Paleoindian projectile point types. What we show here is that this basic design feature also incorporates equally important performance criteria of a technology designed for repeated use and that is resistant to breaking (or durability). This is important because while reducing point thickness increases mechanical advantage, it necessarily reduces robustness, leading to an optimization in the design, where in order for a projectile point to remain a viable component of the hunting toolkit, it must both have the capability to penetrate the prey and remain intact. The allometry of projectile points across types shows evidence of this design optimization in two ways: first, well-known engineering principles state that the allometric ratio of thickness to length must be 2/3rds in order to satisfy the slenderness ratio, thus maximizing the resistance to breaking, and second, in order to maintain optimal penetration performance over multiple uses the width and thickness allometries with volume should be equal, and both have an exponent of 0.29. Both of these predictions are supported by the data. However, there are important differences in the specific designs of different point types, and these undoubtedly result in type-specific differences in penetration capability and robustness.

Figure 10 show the thickness–length and thickness–width allometries by projectile point type, where each type is represented by their average length or thickness as a function of their average volume. By definition, point types that occur above the line in the upper left of the plots have designs that emphasize robustness, whereas types that fall below the line in the lower right have designs that emphasize penetration. For example, Folsom and Midland points show similar allometries in both plots, perhaps unsurprisingly as they are sometimes considered fluted and unfluted versions of each other, and they both fall below the line, suggesting they are designed more for penetration than robustness. It is interesting to note that broken Folsom points are often found in several fragments and may have been designed to fragment on impact, as has been proposed for some triangular points of the Late Prehistoric in eastern North America (Bebber et al. 2017; Engelbrecht 2014, 2015; Mika et al. 2020). Unlike Folsom points, broken Clovis points for example often suffered transverse failures, but the bases often remain intact (Hamilton et al. 2013), suggesting a greater overall robustness, as would be supported by Fig. 10. Particularly robust points include those identified in this database as Harpeth River, but Fig. 10 shows that most Paleoindian point types fall near the boundary and are clearly not randomly distributed in the available 2 dimensions. This suggests that most Paleoindian point types were designed to be general rather than specific, *i.e.*, robust *and* penetrating, rather than robust *or* penetrating. Interestingly, there is no indication in this analysis that Paleoindian hunters carried a range of projectile point types that varied along this spectrum, as often observed in ethnographic hunter–gatherer weaponry where different projectiles are designed for specific prey (Churchill 1993).

Figures 8, 9, and 10 show other interesting details about the relative positions of projectile point types in allometric space. For example, in Fig. 10, the poorly defined collector type "Northumberland" seems to be designed for penetration rather than robustness. Additionally, this type does not consistently co-occur with any other type in the sample in any of the plots (unlike Folsom and Midland), suggesting it has a unique allometric signature. Other notable divergences include Clovis and "Eastern Clovis," a PIDBA category composed of several local projectile point types commonly recognized as "Clovis-like" from eastern North America, including Debert and Vail. However, these two types do not closely co-occur in any of the allometric plots shown here indicating they also have unique allometric signatures rather than clearly belonging to a single type. But importantly, the overall results summarized in Fig. 9 show that these optimal allometries hold across all dimensions and types (with the exception of Hell Gap length allometries). In other words, optimal allometries we show here are properties of individual points, not types.

The allometries we show here hold over a large sample of points that entered the archeological record as complete tools, though at various unknown stages in their use life. Because they are complete (or almost) most were likely lost while still remaining viable, though perhaps some were intentionally discarded for some reason. It is likely that some were used as cutting tools rather than as projectiles, especially later on in their use-life. This seems to be the case for Dalton, for example (Smallwood et al. 2020). Nevertheless, by definition, all of these tools had successfully resisted breaking over their use-life and most likely retained the capability to penetrate prey. Assuming these types reflect real Paleoindian designs that were discrete in time and space this sample is thus an unbiased sample of the range and diversity of projectile points available for use by a Paleoindian hunter within a particular tradition. As such, many if not most of these tools will have been repaired and resharpened and so will have lost mass (*i.e.*, volume

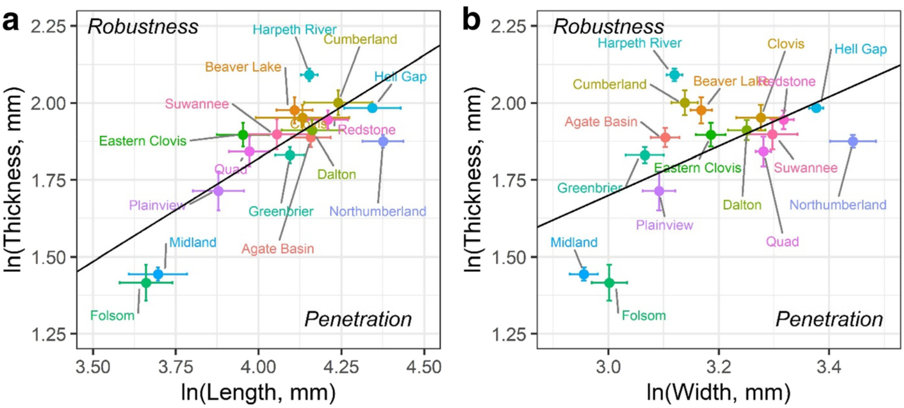

Fig. 10 Bivariate plots of the balance between robustness and penetration across the 16 projectile point types in terms of the allometry of thickness to length (**A**) and width (**B**). The solid lines are the OLS regression slopes of the entire data sets, and the data points are the average values for each projectile point type. Error bars are variances. Point types to the upper left above the line are engineered for robustness, whereas point types to the lower right below the line are engineered for penetration

and weight) from their originally manufactured form. However, the invariance shown here indicates that as points were resharpened and repaired over their use life they had to conform to a well-defined set of engineering constraints (see Fig. 11). In other words, projectile points that did not conform to these criteria were either never manufactured to begin with, or always broke in use and so never entered the archeological record as intact tools. In future studies it would be interesting to compare the allometries we show here with projectile points that entered the archeological either before their use-lives as hunting tools, such as preforms, or bifaces that are argued to have been manufactured for ritual purposes, such as has been suggested for some cached tools, for example (see Kilby 2008; Buchanan et al. 2012a). Similarly, it would be interesting to compare fragmentary to complete points, where possible, in order to understand if broken points have fundamentally different allometries to complete points, and therefore perhaps less robust to failure (Hamilton et al. 2013).

Our results also support the findings of previous studies (*i.e.*, Buchanan et al. 2014; Eren et al. 2014; Loendorf et al. 2018; Tankersley 1994), which show that variation in stone raw material does not play an important role in variation in projectile point dimensions. This suggests raw materials were selected on the basis of physical properties that allowed flintknappers to manufacture specific point forms that were designed to meet a set of performance criteria. There is no evidence to suggest the alternative plausible situation where projectile point form varied in response to the availability of raw materials. This then suggests the lithic economies of Paleoindians allowed enough access to high-quality raw materials to be selective, such that the manufacture and design of points was driven by decisions about engineering and performance (and likely aesthetics). Indeed, recent research highlights the importance of spatially embedded social networks that facilitated the flow of high-quality stone raw materials at multiple scales over late Pleistocene North American landscapes (Buchanan et al. 2016, 2019a, b).

It is also interesting to consider the role of projectile point robustness in the Paleoindian lithic economy. Greater robustness implies greater durability, and thus a

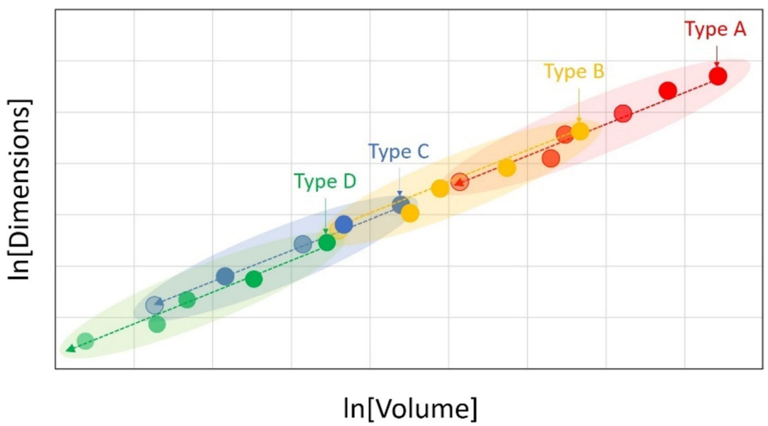

Fig. 11 A schematic representation of the allometry of Paleoindian projectile point design, where different points types of different average sizes are manufactured according to allometric design criteria, and as projectiles are resharpened with each event they necessarily lose mass, volume, and dimension, but along the overall allometric gradient maintaining both robustness and performance

slower turnover in the raw materials used to manufacture projectile points than more fragile points. For types such as Folsom/Midland designed for penetration more than robustness, the implication is that other aspects of the lithic economy must have compensated for higher failure rates further down the line. For example, in order for the Folsom/Midland design to have been a locally optimal solution, flintknappers must have had both access to a sufficient supply of high-quality raw materials but also a production technique that allowed for faster output that compensated for greater loss. Moreover, this design is radically different from the evolutionarily ancestral Clovis design. Clovis points are considerably more robust, produced by biface reduction, and were often deposited in caches. While the historical and geographic relationship between Clovis and Folsom forms is clear, this transition required a major change in the lithic economy, to relatively fragile points produced from flakes that were no longer cached on the landscape. This shift from one local optima to another must have been a conscious innovation that resulted from considering the relative pros and cons of the new design *versus* the old, which then led to a seemingly rapid and widespread transition to Folsom technologies throughout large parts of western North America (Surovell et al. 2016). This would be consistent with previous work where we showed that variation in early Paleoindian point sizes across North America correlated significantly with the body sizes of prey species in different regions (Buchanan et al. 2011). Thus, it is likely that while regional types differentiate from neighboring types as the result of discrete innovations, variation within types over time and space were impacted by drift and copying error (*cf.* Eren et al. 2015; Hamilton and Buchanan 2009).

Clearly, the allometric approach we take here does not capture many of the aspects of variation that are considered in point typology, such as flaking patterns, shape, or the tacit knowledge of the typologist. On the contrary, our dimensional analysis captures only the most rudimentary aspects of the linear dimensions of volume and nothing about the specifics of form. The only way shape is considered here is by statistically examining the fixed effect of point type on allometries, where the type itself is defined by a perceived discreteness in shape, time, and space. Therefore, the statistical effect being measured by the fixed effect is the uniqueness of the criteria used to define the type in the first place. Today, there are increasingly sophisticated ways of quantifying shape using digitization techniques such as geometric morphometrics (*e.g.*, Adams et al. 2013; Slice 2007; Zelditch et al. 2012), and these are becoming standard practice for the study of stone tools (*e.g.*, Archer and Braun 2010; Buchanan et al. 2018, 2020; Charlin and González-José 2012, 2018; Lycett and von Cramon-Taubadel 2013; Lycett et al. 2010; Petřík et al. 2018; Serwatka and Riede 2016; Suárez and Cardillo 2019; Thulman 2019). Undoubtedly, future approaches will use machine learning to capture the deeper subtleties of projectile point typology. However, the strength of the allometric approach is to show how fundamental aspects of design criteria common to all these types are fully described by a consideration of the basic dimensions of volume given the physics of performance, and the optimizations that result.

Finally, while our analyses are not phylogenetic, clearly there is a high retention of design, engineering, and performance criteria throughout the Paleoindian record. Although the geography and timing of the emergence of the Paleoindian projectile point tradition in the New World is unclear, once bifacial point traditions emerge in Paleoindian North America the optimal engineering principles are found quickly, and then retained in descendent forms for millennia. Over time and space, design innovations occur, hence the emergence of Paleoindian point types, but these local innovations occurred within the global design constraints revealed in the optimal allometries shown here.

Code Availability R script available in supplementary materials.

Authors' Contributions BB and MJH contributed equally to the paper.Data AvailabilityData available in supplementary materials.

Compliance with Ethical Standards

Conflicts of Interest The authors declare that they have no conflict of interest.

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